Azonosító jel:

ÉRETTSÉGI VIZSGA • 2007. május 8.

MATEMATIKA
ANGOL NYELVEN
MATHEMATICS

2007. május 8. 8:00

EMELT SZINTŰ
ÍRÁSBELI VIZSGA
ADVANCED LEVEL
WRITTEN EXAM

Az írásbeli vizsga időtartama: 240 perc
The exam is 240 minutes long

Pótlapok száma/Number of extra sheets	
Tisztázati/Final essays	
Piszkozati/Drafts	

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM
MINISTRY OF EDUCATION AND CULTURE

Important information

1. The exam is 240 minutes long, after that you should stop working.

2. You may solve the problems in any order.

3. In Section **II**, you are only required to solve four out of the five problems**. Please remember to enter the number of the question you have not attempted into the empty square below.** Should there *arise any ambiguity* for the examiner as for the question not be marked, it is question no. 9 that will not going to be assessed.

4. You may work with any calculator as long as it is not capable of storing and displaying textual information and you may also consult any type of four digit mathematical table. The use of any other kind of electronic device or written source is forbidden.

5. **Remember to show your reasoning, because a major part of the score is given for this component of your work.**

6. **Remember to outline the substantial calculations**.

7. When you refer to a theorem that has been covered at school and has a common name (e.g. Pithagoras' theorem, sine rule, etc.) you are not expected be state it meticulously; it is usually sufficient to put the name of the theorem. Any reference to any other theorem, however, can be accepted only if it is stated exactly with all the conditions (proof is not required) and you explain how it applies in the given situation.

8. Remember to answer each question (i.e. communicating the result) also in textual form.

9. You are supposed to work in pen; diagrams, however, may also be drawn in pencil. Anything written in pencil outside the diagrams cannot be evaluated by the examiner. Any solution or some part of a solution that is crossed out will not be marked.

10. There is only one solution will be marked for every question. If you attempt a question more than once then you should **clearly indicate** the one to be marked.

11. Please, do not write anything in the shaded rectangular areas.

I.

1. Solve the following pair of simultaneous equations on the set of real numbers.

$$\left.\begin{aligned} \log_2(2x+y)-\log_2(x-1.5y)&=2 \\ \log_3(x+y)+\log_3(x-y)&=2+\log_3 5 \end{aligned}\right\}$$

T.:	11 points	

2. **a)** Sketch the straight lines $y = 0.5x + 2$ and $y = -0.5x + 4$ in the Cartesian coordinate system.

b) The x-axis, the y-axis and the two lines drawn are enclosing a convex quadrilateral. Find the area of this quadrilateral.

c) Four ones among the six points of intersection of the x-axis, the y-axis and the two lines drawn are the vertices of a concave quadrilateral. Find the perimeter of this concave quadrilateral.

a)	**2 points**	
b)	**6 points**	
c)	**5 points**	
T.:	**13 points**	

3. There are six passengers travelling to a scientific conference in a first class compartment of a train to Pécs. Right after departure they realize that there are two of them who happen to know everyone else in the compartment, while each of the remaining four are in aquaintance with exactly four fellow passengers. (Aquaintances are mutual.)

a) Represent the aquaintances on a graph.
b) Upon entering the compartment those in aquaintance are greeting each other by shaking hands. How many handshakes do take place?
c) The six passengers are accomodated in three double-bed rooms. How many ways are there to arrange them if the rooms are not distinguished?

a)	**4 points**	
b)	**3 points**	
c)	**6 points**	
T.:	**13 points**	

4. The edges of the cuboid $ABCDEFGH$ are $AB = 10$, $AD = 8$ and $AE = 6$. The edge vectors starting from A are $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AD} = \mathbf{b}$, $\overrightarrow{AE} = \mathbf{c}$, respectively. Besides these three edge vectors there are three face diagonal vectors and one space diagonal vector also starting from the vertex A. Consider the sum of these seven vectors and denote it by $\overrightarrow{AP}$.

a) Express the vector $\overrightarrow{AP}$ in terms of the edge vectors **a**, **b** and **c**.

b) Find the magnitude of the vector $\overrightarrow{AP}$.

c) Find the angle of the vectors $\overrightarrow{AP}$ and $\overrightarrow{AE}$.

d) Denote the centroid of the triangle HFC by S. Calculate the scalar product $\overrightarrow{AS} \cdot \overrightarrow{AP}$.

a)	**2 points**	
b)	**3 points**	
c)	**3 points**	
d)	**6 points**	
T.:	**14 points**	

II.

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

5. Solve the following equation where p is a real parameter.

$$\frac{x}{x^2-4} + \frac{p}{x^2+2x} + \frac{1}{2x-x^2} = 0$$

Is there any real number p for which the equation has two distinct solutions?
Is there any real number p for which the equation has no solution?

16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

6. Danny has two favourite subjects: maths and biology.

a) One afternoon Danny was counting the fishes in the aquarium of the nearby pet-shop. He has counted *r* big red ones and *s* small striped ones altogether. He did not expose the results to his sister Cathy, however, he provided her with the following information:
"The numbers 4, *r* and *s* in this order are the consecutive terms of a geometric progression while the numbers *r*, *s* and 40 in this order are the consecutive terms of an arithmetic progression."
Find the number of big red fish and also the number of small striped fish Danny has managed to count in the aquarium.

b) In order to settle as many as 100 small fish Danny has purchased a large aquarium. Everything worked out just fine and the livestock has been growing by a steady monthly rate of 20%. At the end of every other month Danny sold the very same percentage of his fish and thus, by the end of the 24^{th} month he was left with 252 fish altogether. What percentage of his fish did Danny sell bimonthly?

c) Cathy received a really nice birthday present from his brother: 20 fishes, 5 big red ones and 15 small striped ones in a spherical aquarium. The kids decided to decorate Cathy's new aquarium with some plants and to do so 8 fishes were selected randomly to be transferred temporarily into a jar.
What is the probability that there were exactly 3 big red ones among the 8 removed fishes?

a)	**5 points**	
b)	**7 points**	
c)	**4 points**	
T.:	**16 points**	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

7. The following table shows the August gross income of the 220 members of staff of a local council.

wages (thousand forints)	68	108	154	184	225
no. of employees	25	65	70	44	16

a) Represent, on a bar chart the distribution of the wages of these 220 employees.
b) Find the mean and the standard deviation of the August wages.
c) What was the average net income in August. (The gross income is 165% of the net income.)
d) In September, the gross income of every single employee is increased by 2 500 forints. How does this influence the standard deviation of their gross income?

a)	**3 points**	
b)	**6 points**	
c)	**3 points**	
d)	**4 points**	
T.:	**16 points**	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

8. The domain of the function f is the interval [0, 5]: $f(x) = 3\cos x - \cos(-x)$.

a) Decide if the following statements are true or false. Justify your answer.
- The function f is bounded.
- The value of x where f admits its minimum and the maximum of f are both irrational numbers.

b) Find the area of the region bounded by the [0, 5] interval of the x-axis; the $[0, f(0)]$ interval of the y-axis; the $[0, f(5)]$ interval of the line $x = 5$ and, finally, the graph of the function f.

a)	**6 points**	
b)	**10 points**	
T.:	**16 points**	

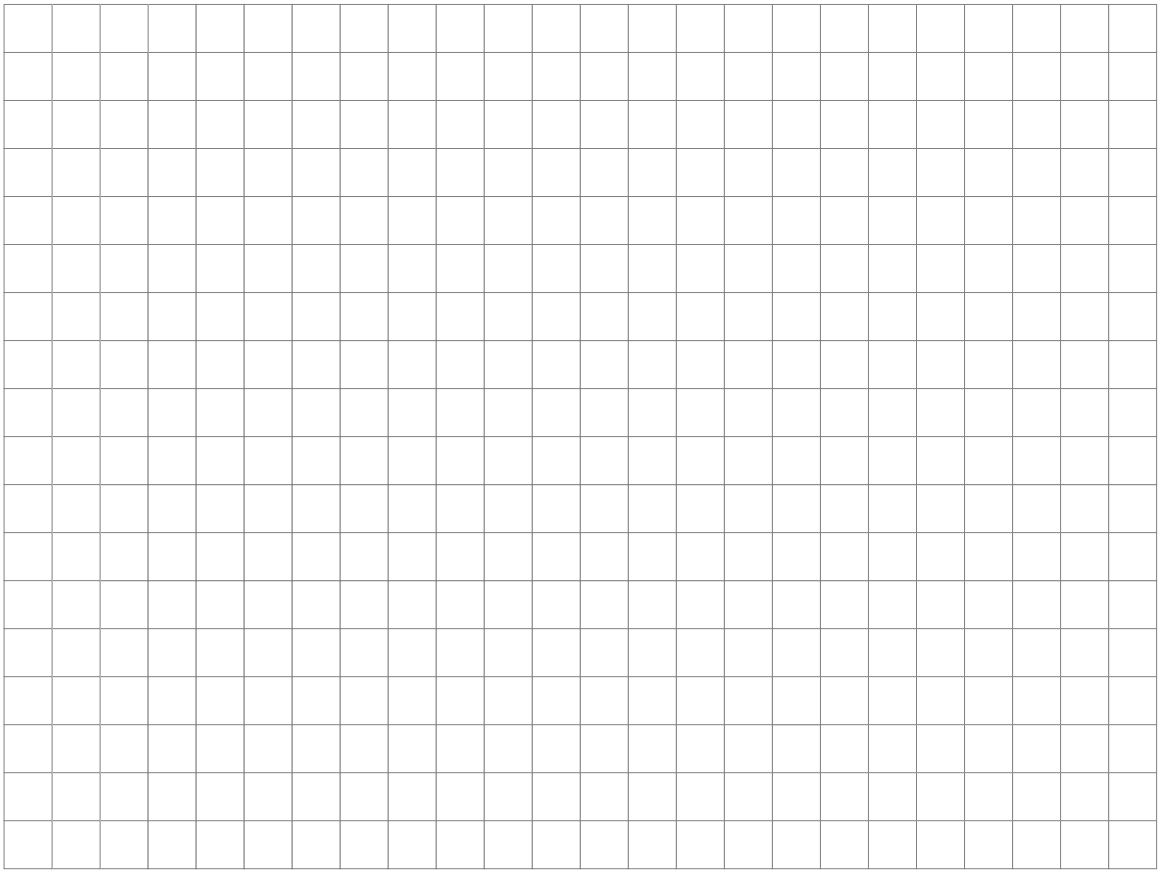
You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

9. Find those two digit positive integers N for which there are exactly two of the following four statements are true and two ones are false.

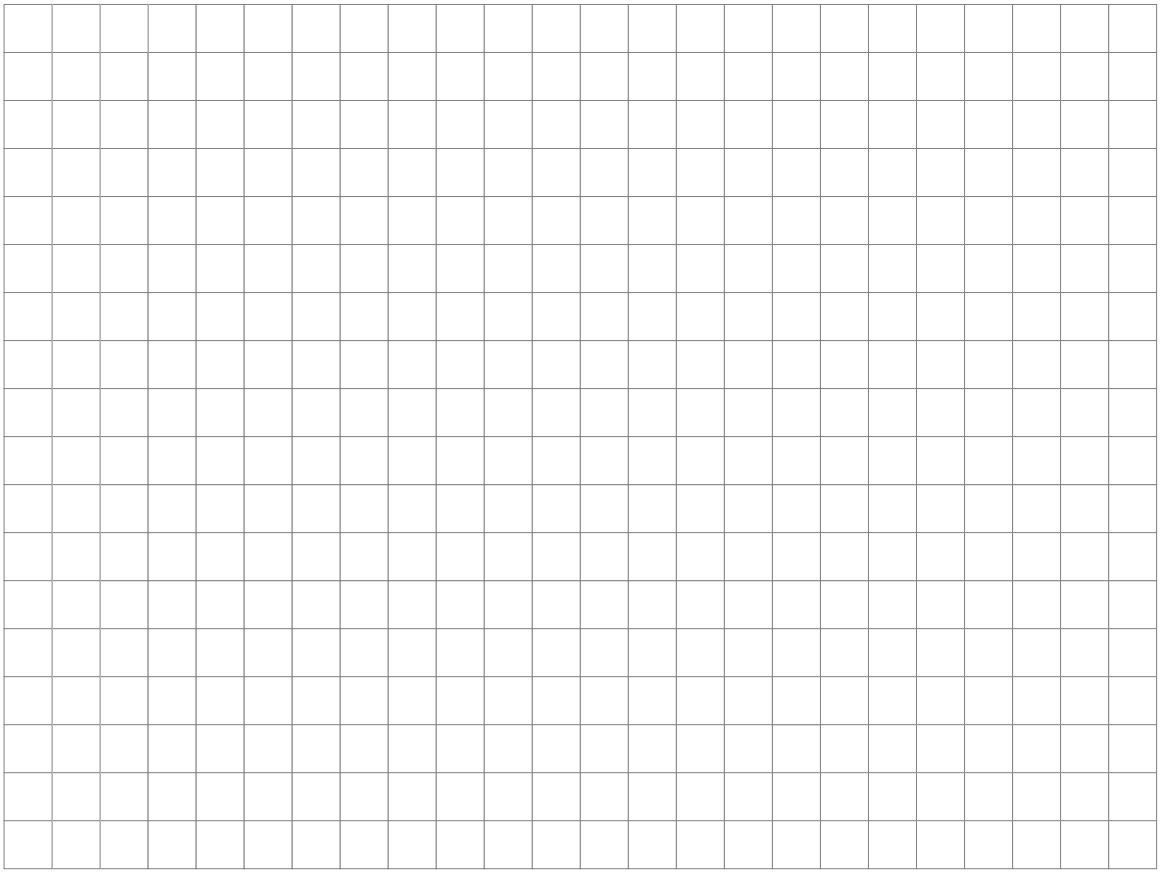
- N is divisible by 7.
- N is divisible by 29.
- $N + 11$ is a perfect square.
- $N - 13$ is a perfect square.

16 points	

(You may also prepare sketches or solutions on this sheet.)



(You may also prepare sketches or solutions on this sheet.)



	number of question	scora	total	maximal score
PART I.	1.			11
	2.			13
	3.			13
	4.			14
PART II.				16
				16
				16
				16
		← problem not chosen		
	TOTAL			**115**

date

examiner

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	a feladat sorszáma/number of question	elért pontszám/score	programba beírt pontszám/ score written in the programme
PART I.	1.		
	2.		
	3.		
	4.		
PART II.			

dátum/date

javító tanár/examiner

jegyző/registrar